647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination

December -2020 (NEW COURSE)

Boolean Algebra & Complex Analysis -1(Core)

Time: 1:30 Hours **Marks: 42**

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

1(A) Answer any two: **(10)**

(1) Define: Equivalence relation. If R is an equivalence relation defined on non-empty set A then prove that:

(i) $b \in [a] \Rightarrow [b] = [a]$

(ii) $[a] = [b] \Leftrightarrow a\ R\ b$

(2) Define: Distributive lattice. Prove that every chain is a distributive Lattice.

(3) If $(L, *, \oplus)$ is a lattice and $a, b \in L$ then prove that $lub\{a, b\} = a \oplus b$ and $glb\{a, b\} = a * b$ with respect to the partial ordering R on L.

(B) Answer any one: **(04)**

(1) Define: (i) Bounded lattice (ii) complemented lattice. Also give an example of a bounded lattice which is not a complemented lattice.

(2) If L_1 be the lattice $D_6 = \{1,2,3,6\}$ (divisor of 6) and L_2 be the lattice $(P(S), \subseteq)$, where $S = \{a, b\}$ then show that D_6 is isomorphic to $P(S)$.

2(A) Answer any two: **(10)**

(1) If $(B, *, \oplus, ', 0, 1)$ is a Boolean Algebra then for any $x_1, x_2 \in B$, Prove that:

(i) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(ii) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(2) State and prove Stone's representation theorem for Boolean algebra.

(3) Define: Boolean expressions. Express $\alpha(x_1, x_2, x_3) = x_1 \oplus x_2$ as the sum of product canonical form.

(B) Answer any one: **(04)**

(1) Define: Minterms and Maxterms, Write all the minterms of the two and three variables.

(2) Obtain the minimal sum of product of the Boolean expression $\alpha(x, y, z) = xyz + xyz' + x'yz' + x'y'z$ by Karnaugh map.

3(A) Answer any two: **(10)**

(1) Obtain C-R conditions for an analytic function $f(z)$ in polar form.

(2) Define: Harmonic function. Show that $u = \frac{1}{2} log(x^2 + y^2)$ is Harmonic function and find its conjugate function.

(3) Find an analytic function $f(z) = u + iv$ such that $u - v = (x - y)(x^2 + 4xy + y^2)$.

(B) Answer any one: **(04)**

(1) Define: Entire function. Prove that $f(z) = e^{-x}(cosy - isiny)$ is an entire function also show that $f'(z) = -f(z)$.

(2) Prove that an analytic function of a constant modulus is also constant in its domain.

4(A) Answer any two: **(10)**

(1) State and prove Cauchy's integral formula.

(2) In usual notation prove that $\left|\int_a^b f(z)dz\right| \leq \int_a^b |f(z)|dz$.

(3) If C is the circle $|z| = R$, where $R > 1$ described in the counter clockwise direction then show that $\left|\int \frac{logz}{z^2} dz\right| < 2\pi\left(\frac{\pi+logR}{R}\right)$.

(B) Answer any one: **(04)**

(1) Define: (i) Jordan arc (ii) Contour. Prove that $\int_0^{1+i} \bar{z}dz = 1$.

(2) Find $\int \frac{z}{(2z-1)(z+1)} dz$, where C: $|z| = 2$

5(A) Answer any two: **(10)**

(1) State the Cauchy's integral formula for derivatives and deduce that $f'(z_0) = \frac{1}{2\pi i}\int \frac{f(z)}{(z-z_0)^2} dz$ and in general $f^n(z_0) = \frac{n!}{2\pi i}\int \frac{f(z)}{(z-z_0)^{n+1}} dz$.

(2) State and prove Morera's theorem.

(3) Prove that $\int \frac{e^{kz}}{z} dz = 2\pi i$, where C: $|z| = 1$ and hence deduce that (i) $\int_0^{\pi} e^{kcos\theta} cos(ksin\theta)\, d\theta = \pi$ (ii) $\int_{-\pi}^{\pi} e^{kcos\theta} sin(ksin\theta)\, d\theta = 0$.

(B) Answer any one: **(04)**

(1) Evaluate: $\int \frac{sin^6 z}{(z-\frac{\pi}{6})^3} dz$, where c: $|z| = 1$.

(2) Evaluate: $\int \frac{e^{2z}}{(z-1)^4} dz$, where C: $|z| = 3$.

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